

Field Redefinition Invariance in Quantum Field Theory

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Abstract

The issue of field redefinition invariance of path integrals in quantum field theory is reexamined. A “paradox” is presented involving the reduction of a $1 + 1$ free massless scalar theory on a Minkowskian cylinder to an effective quantum-mechanical theory. The implementation of field redefinitions both before and after reduction suggests that operator-ordering issues in quantum field theory should not be ignored.

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Field redefinition invariance is a basic property expected of all physically meaningful quantities, such as the poles of renormalized propagators. By contrast, there exist quantities related to the specific choice of field variables, such as wave function renormalization factors, whose values depend on the particular parametrization.

The question then arises as to the transformation properties of functional integrals under nonlinear point canonical transformations. For the quantum-mechanical counterpart of this problem, additional terms $O(\hbar^2)$ appear in the path integral [1–3]. This phenomenon—which can be viewed as a manifestation of the stochastic nature of the Lagrangian formulation of the path integral—is usually studied by introducing a discretization of the time variable [4–6]. The ensuing “extra” terms are an inevitable consequence of the quantization of the theory, which promotes the classical coordinates to quantum operators; in effect, for every operator ordering of the associated Hamiltonian,¹ there exists a particular prescription for handling the lattice definition of the path integral [3].

The standard lore in quantum field theory dictates, in contradistinction to the quantum-mechanical procedure, that no additional terms are needed. More precisely, the action is assumed to change by direct substitution of the field transformation, together with the addition of the Jacobian determinant associated with the change of field variables [7]. Moreover, the Jacobian becomes superfluous within the dimensional-regularization scheme—upon exponentiation, the formally infinite spacetime delta function $\delta^{(d)}(0)$ generated by the trace is set equal to zero [8]. A similar line of reasoning is employed to argue away any possible contribution by “extra” terms generated in the path integral; these terms—being a manifestation of operator ordering—would vanish by dimensional regularization, because they would involve delta functions at zero spatial argument, as follows from $[\Phi(x), \Pi(x)] = i\hbar\delta^{(d-1)}(0)$. More precisely, the standard justification for this procedure is based on the assumed existence and necessity of local counterterms in the action, so that Jacobians and any other

¹In this Letter we adopt Weyl ordering, which corresponds to the midpoint prescription.

additional terms resulting from operator ordering (all of which are local quantities in the action), have the only effect of changing the coefficients of these local counterterms.

However, upon closer examination, one realizes that a solid justification for setting infinite quantities equal to zero is still lacking. Even if the validity of dimensional regularization is not questioned, one could analyze the problem from the lattice point of view,² in which the Jacobian as well as the “extra” terms, do not vanish.³

The purpose of this Letter is to investigate these questions in a field theory toy model, in which we have full control of regularization issues and can test the relevance of non-linear field redefinitions. Further technical details will appear elsewhere [9]. Our model is a free massless scalar quantum field theory in $1 + 1$ dimensions on a flat Minkowskian spacetime characterized by the action

$$S[\Phi] = \frac{1}{2} \int_{\mathbb{R}^1 \times S^1} d^2x \, \eta^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi = \frac{1}{2} \int_{\mathbb{R}^1} dt \int_{S^1} dx \left\{ [\dot{\Phi}(t, x)]^2 - [\partial_x \Phi(t, x)]^2 \right\} , \quad (1)$$

where the conventions $x^0 = t$, $x^1 = x$, and $\eta_{00} = 1$, $\eta_{11} = -1$ are chosen; $\mathbb{R}^1 \times S^1$ stands for a Minkowskian cylinder, with the spatial coordinate subject to the identification $x \cong x + L$ (where $L = 2\pi R$, with R being the radius of this compactified one-dimensional space)—explicitly, the scalar field is subject to the periodicity condition $\Phi(t, x + L) = \Phi(t, x)$. Our approach is based on making a nonlinear field redefinition

$$\Phi = F[\tilde{\Phi}] , \quad (2)$$

using two different methods and comparing the corresponding results. In Method 1, the compactified spatial coordinate is integrated out to yield an effective quantum-mechanical

²The spacetime delta function at zero argument is related to the inverse lattice spacing a , in the form $\delta^{(d)}(0) \sim a^{-d}$.

³Not surprisingly, dimensional regularization does fail for theories in which the action has some feature depending on the dimensionality of spacetime; for example, for theories with chiral fermions, conformal symmetries, and with nontrivial spacetime topology.

problem, which is then subject to the quantum-mechanical counterpart of the transformation (2), for which the existence of an “extra” term is a well-established result [6]. In Method 2, the standard quantum-field theoretical lore is applied directly to the field redefinition (2) of the 1+1-dimensional field theory, followed by a reduction to a quantum-mechanical action by integrating out the spatial coordinate. The required identity of the results of the two methods leads to a remarkable “paradox”: no new terms are developed in the field-theory case (Method 2), despite the appearance of “extra” terms for the quantum-mechanical case (Method 1). Reconciling these two methods calls for either a detailed explanation or a revision of the standard lore.

A digression is in order for subsequent notational and computational purposes. In our Letter we will use the ultralocal spacetime metric [9]

$$\mathcal{G}[\tilde{\Phi}](t, x; t', x') = \delta(t - t') \delta(x - x') \left(F'[\tilde{\Phi}(t, x)] \right)^2 = \delta(t - t') g[\tilde{\Phi}] , \quad (3)$$

where the reduced metric $g[\tilde{\Phi}]$ will be useful for the calculations of Method 1. When implementing the nonlinear field redefinition (2), the transformed action $S[\tilde{\Phi}]$ has two (Method 2) or three (Method 1) pieces: (i) the part obtained by direct substitution into the original free action (1), $S_0[\tilde{\Phi}] = S[F[\tilde{\Phi}]]$; (ii) the effective action arising from exponentiation of the Jacobian,

$$S_{\text{Jacobian}}[\tilde{\Phi}] = -\frac{i\hbar}{2} \text{Tr} \ln \mathcal{G}[\tilde{\Phi}] , \quad (4)$$

where Tr stands for the spacetime trace; and (iii) the “extra” term $S_{\text{extra}}[\tilde{\Phi}]$ of $O(\hbar^2)$ arising from its quantum-mechanical Weyl-ordered counterpart [6] (for Method 1).

Method 1.

Introducing the formal inner product

$$\langle \Phi, \Psi \rangle(t) = \int_{S^1} dx \Phi(t, x) \Psi(t, x) , \quad (5)$$

the action (1) becomes

$$S[\Phi] = \frac{1}{2} \int_{\mathbb{R}^1} dt \left[\langle \dot{\Phi}, \dot{\Phi} \rangle(t) - \langle \partial_x \Phi, \partial_x \Phi \rangle(t) \right] , \quad (6)$$

which may be converted into an effective quantum-mechanical problem by expanding the scalar field in periodic eigenfunctions

$$\Phi(t, x) = \sum_{p \in \mathbb{Z}} \phi^p(t) b_p(x) \quad (7)$$

(Kaluza-Klein-like decomposition) and integrating out the x dependence. In Eq. (7), $\{b_p(x)\}_{p \in \mathbb{Z}}$ is a basis for the space of real functions on the interval $x \in [-L/2, L/2]$ (or any other interval of length L), with periodic boundary conditions; even though it is customary to use the Fourier basis $b_p(x) = e^{2\pi i p x / L}$, our analysis will be carried for an arbitrary $\{b_p(x)\}$.

In order to avoid the appearance of awkward divergences, we will work with the discrete version of the theory, as defined on a Minkowskian spacetime cylindrical lattice. By first considering a finite time interval T , the lattice spacings become $\epsilon_0 = T/N_0$ and $\epsilon_1 = L/N_1$, where the large integers N_0 and N_1 are chosen to be even for the sake of convenience. Then, the discrete points (t_α, x_j) on the lattice are defined by $t_\alpha = \alpha \epsilon_0$ and $x_j = j \epsilon_1$, where $\alpha \in I_{N_0}$ and $j \in I_{N_1}$ are selected from the basic sets of the form $I_N = \{-N/2 + 1, \dots, N/2\}$. Introducing the notation $\varphi^j(t) = \Phi(t, x_j)$, the lattice action becomes

$$S[\varphi] = \frac{1}{2} \epsilon_0 \epsilon_1 \sum_{\alpha \in I_{N_0}} \sum_{j \in I_{N_1}} \left\{ \left[\frac{\varphi^j(t_{\alpha+1}) - \varphi^j(t_\alpha)}{\epsilon_0} \right]^2 - \left[\frac{\varphi^{j+1}(t_\alpha) - \varphi^j(t_\alpha)}{\epsilon_1} \right]^2 \right\}. \quad (8)$$

In this Letter, we will focus on the discretization of the spatial variable, as a way of introducing a quantum-mechanical system with a finite number of degrees of freedom. Instead, the time variable will be kept continuous, with the understanding that discretization of the time—independently from x —can be implemented whenever this proves convenient.⁴ Then, we will simplify the notation with the identifications $N \equiv N_1$ and $\epsilon \equiv \epsilon_1$. Notice that, in the limit $N \rightarrow \infty$, the spatial lattice generates the continuous interval $x \in [-L/2, L/2]$.

⁴In Ref. [6], it was shown that the use of a lattice for the variable t (with an independent lattice constant ϵ_0 arising from an even number N_0 of points) permits the correct quantum-mechanical treatment of “extra” terms $O(\hbar^2)$ in the limit $N_0 \rightarrow \infty$, under a nonlinear change of variables.

The main advantage of introducing a lattice for our problem lies in that it eliminates ultraviolet divergences, by reducing the space of functions defined on S^1 to a finite-dimensional space \mathcal{V}_N . In \mathcal{V}_N , an arbitrary basis $\{b_p\}_{p \in I_N}$ can be chosen by selecting N linearly independent vectors $(b_p(x_j))_{j \in I_N}$ (for $p \in I_N$). Then, for any field variable,

$$\Phi(t, x) = \sum_{p \in I_N} \phi^p(t) b_p(x) , \quad (9)$$

which reproduces the Kaluza-Klein decomposition (7) in the continuum limit, while

$$\varphi^j(t) = \sum_{p \in I_N} \phi^p(t) \Lambda_p^j , \quad (10)$$

with $\Lambda_p^j = b_p(x_j)$ defining an invertible $N \times N$ matrix. A particular convenient choice, in addition to the Fourier basis, is provided by the “canonical” basis $c_j(x)$, which is defined by $c_j(x_k) = \delta_j^k$ and amounts to a real-space lattice representation of the field $\Phi(t, x)$, with components $\varphi^j(t)$. The discrete version of the inner product (5),

$$\langle \Phi, \Psi \rangle(t) = \frac{L}{N} \sum_{j \in I_N} \varphi^j(t) \psi^j(t) = \sum_{p, q \in I_N} G_{pq} \phi^p \psi^q \quad (11)$$

defines the basic symmetric metric $G_{pq} = \langle b_p, b_q \rangle$, in terms of which the resulting action is

$$S[\phi] = \int dt \left[\sum_{p, q \in I_N} \frac{1}{2} g_{pq}[\phi] \dot{\phi}^p \dot{\phi}^q - \frac{1}{2} h_{pq} \phi^p \phi^q \right] \quad (12)$$

[cf. Eq. (6)], where the matrix elements $g_{pq}[\phi]$ (metric) and h_{pq} , associated with the time and spatial derivatives respectively, admit the expressions

$$g[\phi] = G \quad ; \quad h = \Delta^T G \Delta . \quad (13)$$

In Eq. (13) the elements of the matrix Δ are defined as the components of the lattice counterparts of the spatial derivatives $b'_p(x)$, i.e., $b_p(x_{j+1}) - b_p(x_j) = \epsilon \sum_{r \in I_N} \Delta_p^r b_r(x_j)$. The matrix Δ is explicitly dependent on ϵ or $1/N$, i.e., $\Delta = \Delta(1/N)$; however, it admits the asymptotic expansion $\Delta(1/N) = \Delta(0) + O(1/N)$, so that a definite finite value $\Delta^{(0)} = \Delta(0)$ exists in the continuum limit ($N \rightarrow \infty$). This limit will be eventually assumed in Eq. (12)

and similar expressions, in which case the substitution $I_N \rightarrow Z$ should be performed.⁵

Let us now consider the field redefinition (2) and expand the new field $\tilde{\Phi}(t, x)$ in modes,

$$\tilde{\Phi}(t, x) = \sum_{p \in I_N} \tilde{\phi}^p(t) b_p(x) , \quad (14)$$

with the implicit transformation

$$\phi^p \equiv f^p[\tilde{\phi}] . \quad (15)$$

Then, the reduced metric $g[\tilde{\Phi}]$ (with respect to the new coordinates), as defined in Eq. (3) from the ultralocal full-fledged metric $\mathcal{G}[\tilde{\Phi}](t, x; t', x')$, is diagonal; in fact, the lattice version of Eq. (3) implies that the reduced lattice metric coefficients are given from $g_{jk}[\tilde{\varphi}]$, i.e.,

$$g[\tilde{\varphi}] = \frac{L}{N} \text{diag} \left\{ (F'[\tilde{\varphi}])^2 \right\} . \quad (16)$$

On the other hand, with respect to any other basis,

$$g[\tilde{\phi}] = \Lambda^T g[\tilde{\varphi}] \Lambda , \quad (17)$$

where $\Lambda_p^j = \partial \tilde{\varphi}^j / \partial \tilde{\phi}^p = b_p(x_j)$.

The change of variables (15) in the quantum-mechanical path integral should be implemented by including the “extra” term, i.e., the transformed action $S[\tilde{\phi}]$ is

$$S[\tilde{\phi}] = S_0[\tilde{\phi}] + S_{\text{Jacobian}}[\tilde{\phi}] + S_{\text{extra}}[\tilde{\phi}] . \quad (18)$$

The first term in Eq. (18) can be computed by direct substitution in Eq. (12),

$$S_0[\tilde{\phi}] = S[f[\tilde{\phi}]] = \int dt \left[\frac{1}{2} \sum_{p, q \in I_N} g_{pq}[\tilde{\phi}] \dot{\tilde{\phi}}^p \dot{\tilde{\phi}}^q - \frac{1}{2} h_{pq} f^p[\tilde{\phi}] f^q[\tilde{\phi}] \right] . \quad (19)$$

⁵Parenthetically, Eq. (12) describes a system of N coupled quantum-mechanical oscillators $\phi^p(t)$; for example, when the Fourier basis $b_p(x) = e^{2\pi i p x / L}$ is chosen, then $g_{pq}[\phi] = L \delta_{p, -q}$ and $\Delta_q^p = \frac{2\pi i p}{L} \delta_q^p + O(1/N)$, whence Eq. (12) provides the frequencies $\omega_p = 2\pi p / L$, as $N \rightarrow \infty$.

In the quantum-mechanical path integral, the Jacobian determinant, $\prod_{\alpha \in I_{N_0}} \left\{ \left(g[\tilde{\phi}](t_\alpha) \right)^{1/2} \right\}$, leads to the standard contribution to the action,

$$S_{\text{Jacobian}}[\tilde{\phi}] = -\frac{i\hbar}{2} \sum_{\alpha \in I_{N_0}} \text{tr} \ln \left\{ g[\tilde{\phi}](t_\alpha) \right\} = -\frac{i\hbar}{2} \text{Tr} \ln \left\{ g[\tilde{\phi}] \delta(t - t') \right\} \quad (20)$$

[cf. Eq. (4)], where tr stands for the reduced spatial trace (with respect to spatial indices alone), as opposed to the spacetime trace Tr .

Finally, the “extra” term arising from the stochastic nature of the path integral is⁶

$$S_{\text{extra}}[\tilde{\phi}] = -\frac{\hbar^2}{8} \int dt g^{pq}[\tilde{\phi}] \Gamma_{rp}^s[\tilde{\phi}] \Gamma_{sq}^r[\tilde{\phi}] = -\frac{\hbar^2}{8} \int dt \text{tr} \left(g^{-1}[\tilde{\phi}] \Xi[\tilde{\phi}] \right), \quad (21)$$

where $\Gamma_{rp}^s[\tilde{\phi}]$ are the connection coefficients associated with the metric $g_{pq}[\tilde{\phi}]$ and $\Xi_{pq} = \Gamma_{rp}^s \Gamma_{sq}^r$. Equation (21) requires the evaluation of $\text{tr} \left(g^{-1}[\tilde{\phi}] \Xi[\tilde{\phi}] \right)$, which can be performed in an arbitrary basis $\{b_p\}$, due to the tensor nature of the expressions involved in the lattice version of the theory. However, this is most easily done in real space, where the metric is diagonal⁷ [Eq. (16)], so that the inverse metric is $g^{-1}[\tilde{\phi}] = N L^{-1} \text{diag} \left\{ (F'[\tilde{\phi}])^{-2} \right\}$ and the connection coefficients are $\Gamma[\tilde{\phi}] = \text{diag}^{(3)} \left\{ F''[\tilde{\phi}] / F'[\tilde{\phi}] \right\}$ (diagonal with respect to the three indices in real space). Then,

$$\text{tr} \left(g^{-1}[\tilde{\phi}] \Xi[\tilde{\phi}] \right) = \frac{N}{L} \sum_{j \in I_N} \left[\frac{(F''[\tilde{\phi}])^2}{(F'[\tilde{\phi}])^4} \right]_j. \quad (22)$$

For later comparison with Method 2, it is useful to rewrite Eqs. (21) and (22) explicitly in terms of the field $\tilde{\Phi}$; then,

$$S_{\text{extra}}[\tilde{\Phi}] = -\frac{\hbar^2}{8} \left(\frac{N}{L} \right)^2 \int d^2x \frac{(F''[\tilde{\Phi}])^2}{(F'[\tilde{\Phi}])^4}. \quad (23)$$

⁶The Einstein summation convention for repeated indices is adopted from Eq. (21) on.

⁷Obviously, when the metric is nondiagonal, the computations are quite a bit lengthier. For example, for the Fourier basis of exponentials and $\Phi = \tilde{\Phi} + \alpha \tilde{\Phi}^\nu$, the same results follow straightforwardly from $g[\tilde{\phi}] = L N^{-1} [1 + \alpha \nu (\tilde{\Phi})^{\nu-1}]^2$, with the matrix $(\tilde{\Phi})_q^p = \tilde{\phi}^{p-q}$ [9].

Method 2.

In this method, the field redefinition (2) is applied first, while the expansion in Kaluza-Klein modes is later performed in the transformed field theory. The terms in the action obtained from Method 2 will be written with hats to distinguish them from those of Method 1. By direct substitution of the field redefinition (2) in the action (1), the piece

$$\hat{S}_0[\tilde{\Phi}] = S[F[\tilde{\Phi}]] = \frac{1}{2} \int d^2x \, \eta^{\mu\nu} \left(F'[\tilde{\Phi}] \right)^2 \partial_\mu \tilde{\Phi} \partial_\nu \tilde{\Phi} \quad (24)$$

develops derivative interaction terms, while the Jacobian, from Eqs. (3) and (4), yields

$$\hat{S}_{\text{Jacobian}}[\tilde{\Phi}] = -i\hbar \delta^{(2)}(0) \int d^2x \ln F'[\tilde{\Phi}] . \quad (25)$$

The conventional arguments within the standard lore would imply that the total action is given by only these two contributions, $\hat{S}[\tilde{\Phi}] = \hat{S}_0[\tilde{\Phi}] + \hat{S}_{\text{Jacobian}}[\tilde{\Phi}]$.

Finally, the action $\hat{S}[\tilde{\Phi}]$ can be converted into an effective quantum-mechanical one, $\hat{S}[\tilde{\phi}] = \hat{S}_0[\tilde{\phi}] + \hat{S}_{\text{Jacobian}}[\tilde{\phi}]$, by integrating out the spatial coordinate, as follows:

$$\hat{S}_0[\tilde{\phi}] = \int dt \sum_{p,q \in Z} \left[\frac{1}{2} \hat{g}_{pq}[\tilde{\phi}] \dot{\tilde{\phi}}^p \dot{\tilde{\phi}}^q - \frac{1}{2} \hat{h}_{pq} f^p[\tilde{\phi}] f^q[\tilde{\phi}] \right] \quad (26)$$

and

$$\hat{S}_{\text{Jacobian}}[\tilde{\phi}] = -\frac{i\hbar}{2} \text{Tr} \ln \left\{ g[\tilde{\phi}] \delta(t - t') \right\} . \quad (27)$$

In Eq. (26),

$$\hat{g}_{pq}[\tilde{\phi}] = \int dx \, b_p(x) b_q(x) \left(F'[\tilde{\Phi}(x)] \right)^2 = g_{pq}[\tilde{\phi}] , \quad (28)$$

as follows from the limit $N \rightarrow \infty$ of Eqs. (16) and (17); likewise $\hat{h}_{pq} = h_{pq}$, so that $\hat{g}_{pq}[\tilde{\phi}]$ and \hat{h}_{pq} coincide with the corresponding matrix elements appearing in the continuum limit of the quantum-mechanical version of this calculation.

Comparison of Methods.

The transformations involved in Methods 1 and 2 are represented in the diagram

$$\begin{array}{ccc}
S[\Phi] & \xrightarrow{\Phi = F[\tilde{\Phi}]} & \hat{S}[\tilde{\Phi}] \\
\mathcal{R} \downarrow & & \mathcal{R} \downarrow \\
S[\phi] & \xrightarrow{\phi = f[\tilde{\phi}]} & S[\tilde{\phi}] \stackrel{?}{=} \hat{S}[\tilde{\phi}]
\end{array} , \tag{29}$$

where \mathcal{R} stands for reduction to a quantum-mechanical problem (by integrating out the spatial coordinate). The identity of the results of the two methods amounts to the equality of the two actions for the effective quantum-mechanical theory; in other words, it is equivalent to the statement that diagram (29) be commutative. However, if the standard lore holds true, the action $\hat{S}[\tilde{\phi}]$ lacks the “extra” terms, so that

$$S_0[\tilde{\phi}] + S_{\text{Jacobian}}[\tilde{\phi}] + S_{\text{extra}}[\tilde{\phi}] = \hat{S}_0[\tilde{\phi}] + \hat{S}_{\text{Jacobian}}[\tilde{\phi}] . \tag{30}$$

Let us now analyze the feasibility of Eq. (30). Firstly, the equality $S_0[\tilde{\phi}] = \hat{S}_0[\tilde{\phi}]$ follows from Eqs. (19) and (26). Secondly, the equality of the Jacobian factors, $S_{\text{Jacobian}}[\tilde{\phi}] = \hat{S}_{\text{Jacobian}}[\tilde{\phi}]$ is seen from Eqs. (20) and (27). Finally, due to the identity of the first two terms, it is clear that Eq. (30) is incompatible with the existence of a nonzero term $S_{\text{extra}}[\tilde{\phi}]$. In other words, we are now confronted with the central issue of this Letter: in Method 1, $S_{\text{extra}}[\tilde{\phi}] \neq 0$, while in Method 2, the standard rules for nonlinear field redefinitions failed to generate such a term. The inescapable conclusion, if $S_{\text{extra}}[\tilde{\phi}]$ cannot be rationalized to vanish, is that this term should have emerged at the level of quantum field theory from the nonlinear field redefinition. Therefore, from Eq. (23) and the identification

$$\frac{N}{L} = \delta(x=0) \tag{31}$$

(in the limit $N \rightarrow \infty$)—which is recognized to be the standard condition for the transition from the lattice version of the theory to its continuous counterpart—it follows that the final expression for the “extra” quantum-field theoretical term is

$$S_{\text{extra}}[\tilde{\Phi}] = -\frac{\hbar^2}{8} [\delta(x=0)]^2 \int d^2x \frac{(F''[\tilde{\Phi}])^2}{(F'[\tilde{\Phi}])^4}. \quad (32)$$

This divergent term is proportional to the square of the spatial delta function rather than the two-dimensional delta function at zero argument. An analogue of this result was found in the early literature on four-dimensional chiral dynamics [10–12]. In fact, in d -dimensional field theory, a term proportional to $[\delta^{(d-1)}(0)]^2$ appears [14].

A final remark is in order. An alternative to the approach of Method 2 is afforded by the addition, after field redefinition, of an infinite series of counterterms,

$$S[\tilde{\Phi}] = \frac{1}{2} \int d^2x \, \eta^{\mu\nu} (F'[\tilde{\Phi}])^2 \partial_\mu \tilde{\Phi} \partial_\nu \tilde{\Phi} - i\hbar \delta^{(2)}(0) \int d^2x \ln F'[\tilde{\Phi}] + \int d^2x \sum_{\ell=1}^{\infty} c_\ell \tilde{\Phi}^\ell, \quad (33)$$

where the unknown coefficients c_ℓ can be evaluated by computing physically significant quantities and matching with the original free theory. The advantage of our approach lies in that we have been able to directly derive the simple expression in Eq. (32), which would otherwise be obtained by laboriously computing Feynman diagrams and summing the series in Eq. (33).

In conclusion, we have shown evidence for a single “extra” term being generated in the 1+1 dimensional quantum field theory on a Minkowskian cylinder upon making a nonlinear field redefinition. An extension of the work in quantum mechanics [6], as well as a perturbative calculation based upon these results, gives preliminary confirmation of the existence of “extra” terms in quantum field theory, at least for the case of flat Euclidean d -dimensional spacetime. This work also reveals the need for a more careful use of dimensional regularization in higher-order calculations. Details will appear elsewhere [14].

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